2024-2025
Fall Semester

Course of Power System Analysis
Thermal phenomena

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Outline

Thermal phenomena in lines

Criteria for steady-state conductor sizing

Maximum current for bare conductors

Maximum current for cables in steady-state conditions

Adiabatic thermal phenomena in the event of a short-circuit

Permissible cable temperature in the event of a short-circuit

Thermal phenomena in lines

Electric lines carrying an electric current give rise to **Joule effect losses** $W_j=RI^2$.

These losses, which will transform into **heat**, are:

1. In part **stored in the conductors themselves**
2. In part **released to the environment**.

The primary effect of these losses is the **increase in the temperature of the conductors.**

Problem: define maximum current or short-circuit/overload behavior for transmission lines (note that similar considerations apply to other devices like transformers, synchronous machines, induction machines etc.).

Thermal phenomena in lines

Hypothesis: conductor of electrical resistance R with a **homogeneous conductivity**, through which an RMS electric current I flows.

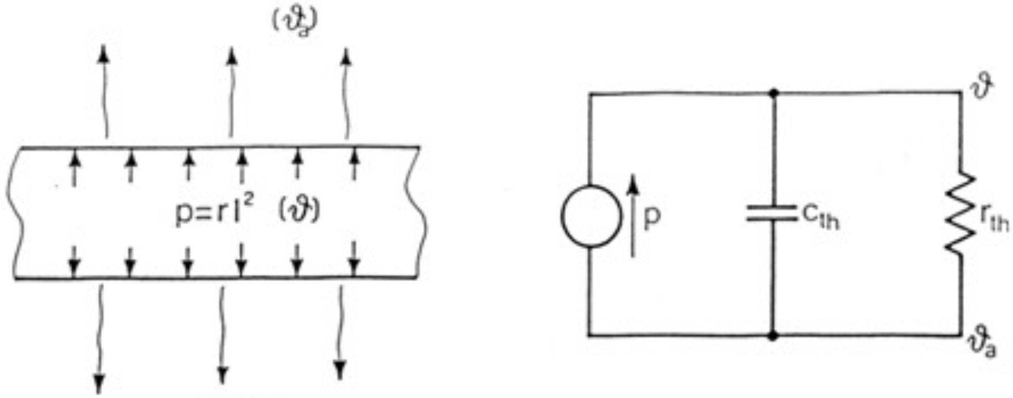
Heat balance equation:

$$RI^2 dt = \gamma v c \, d\theta + K_t S_d (\theta - \theta_a) dt$$

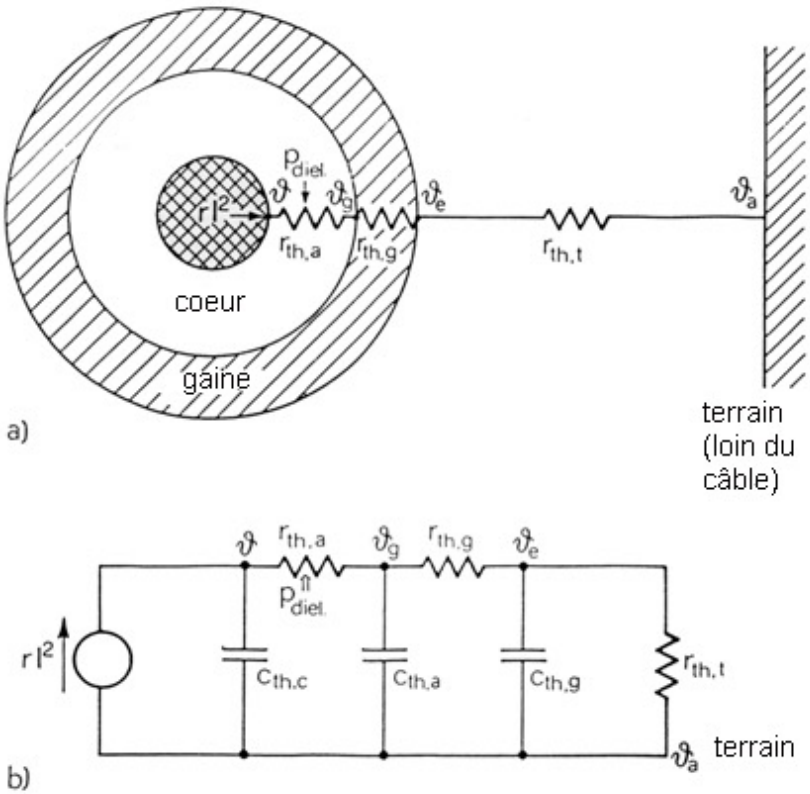
where:

- θ is the temperature of the conductor (°C);
- γ is the density (kg/m^3);
- v is the volume (m^3);
- c is the specific heat (J/kg°C);
- K_t is the global thermal heat transfer coefficient (W/m^2°C);
- S_d is the heat exchange surface area (m^2).

Thermal phenomena in lines



Heat release from a **bare conductor** in air heated by thermal power p and its equivalent thermal circuit.



Schematic representation of heat transfer for a **single-core cable with insulation laid in homogeneous soil** (a) and its equivalent thermal circuit (b).

Thermal phenomena in lines

Hypothesis: all of the enumerated **coefficients** of the energy balance are **constant.**

We also consider that at $t=0$, the **temperature $\theta = \theta_a$ is the ambient temperature** (held constant), such that the above differential equation can be integrated as follows from the associated homogeneous equation:

$$\gamma v c \; d\theta + K_t S_d (\theta - \theta_a) dt = 0 \rightarrow \frac{d\theta}{\Delta\theta} = -\frac{K_t S_d}{\gamma v c} dt$$

$$\Delta\theta(t) = A e^{-\frac{t}{\tau}}$$

where:
$\Delta\theta = \theta - \theta_a$; A is the integration constant and $\tau = \dfrac{\gamma v c}{K_t S_d}$ is the time constant.

Thermal phenomena in lines

The particular integral can be obtained by **solving the equation at steady-state, namely for** $t \to \infty$, i.e., when the system reaches the thermal regime:

$$\Delta\theta(t) = \frac{RI^2}{K_t S_d}$$

The general solution is:

$$\Delta\theta(t) = \frac{RI^2}{K_t S_d} + Ae^{-\frac{t}{\tau}}$$

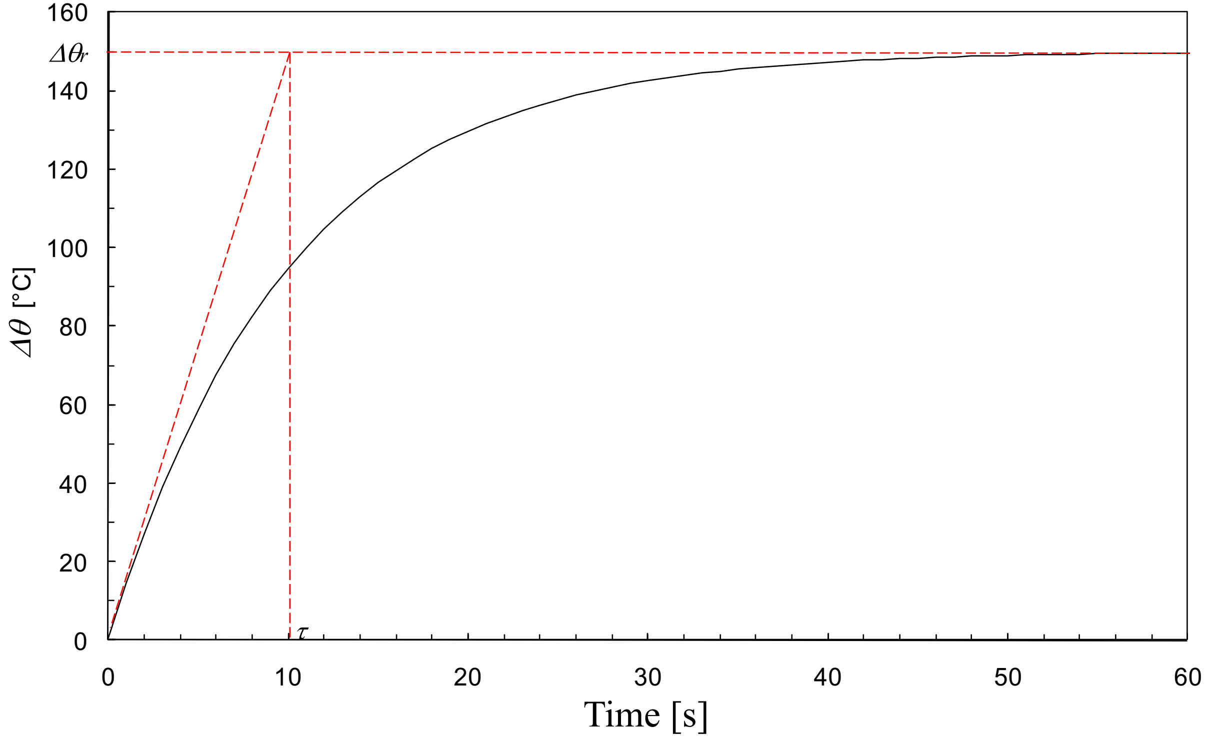
The **integration constant** A can be determined using the **initial conditions** $t=0$, $\Delta\theta=0$, for which we obtain

$$A = -\frac{RI^2}{K_t S_d} \to \Delta\theta(t) = \frac{RI^2}{K_t S_d}\left(1 - e^{-\frac{t}{\tau}}\right)$$

Thermal phenomena in lines

For $t \to \infty$, we have $\Delta\theta_r = \theta_r - \theta_a$ (where θ_r is the steady-state temperature and θ_a is the ambient temperature). The preceding equation becomes:

$$\Delta\theta(t) = \Delta\theta_r \left(1 - e^{-\frac{t}{\tau}} \right)$$



Thermal phenomena in lines

If the heat source is **removed** (i.e., the electric current is interrupted), the device under examination will cool down from the initial temperature θ_i. The initial temperature difference at $t=0$, relative to ambient temperature, will be $\Delta\theta_i=\theta_i-\theta_a$ and the differential equation driving the phenomenon is as follows:

$$\gamma v c \frac{d\theta}{dt} + K_t S_d(\theta - \theta_a) = 0$$

this is the **homogeneous differential equation of the initial heat balance equation**.

The solution, with the hypothesis of $\Delta\theta=\theta-\theta_a$ and $\Delta\theta_i=\theta_i-\theta_a$, is as follows:

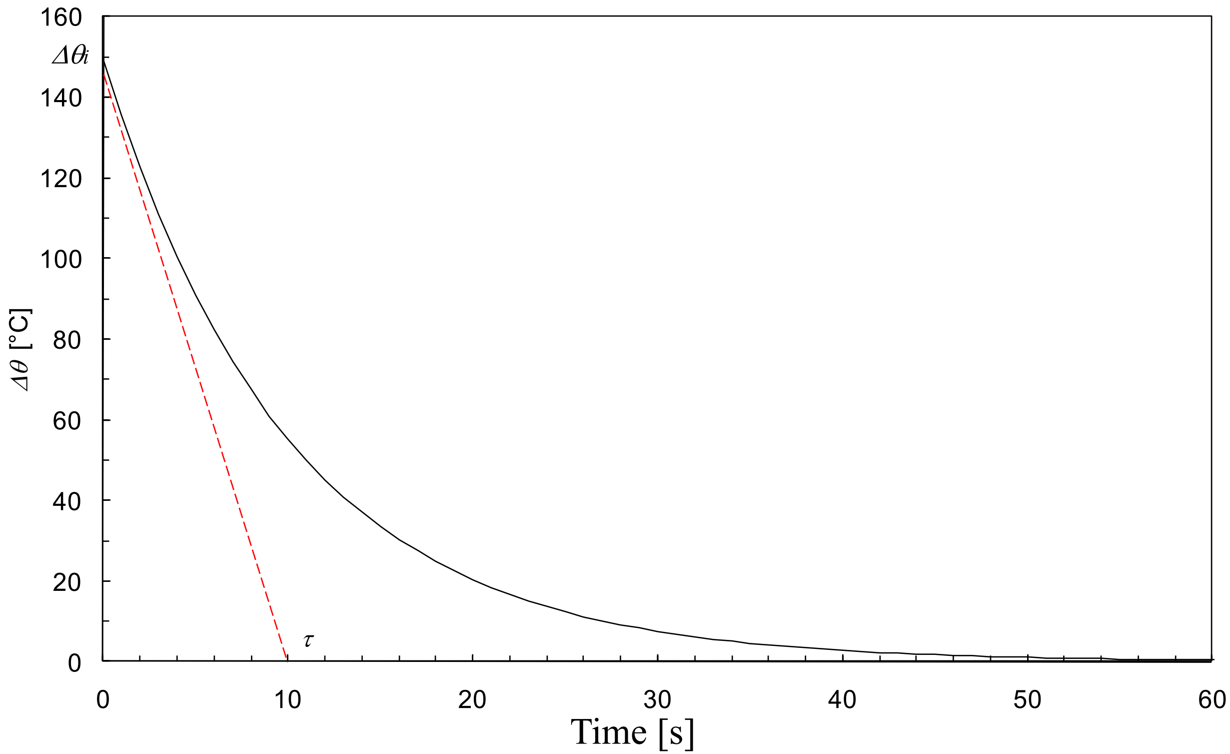
$$\Delta\theta(t) = \Delta\theta_i e^{-\frac{t}{\tau}}$$

Thermal phenomena in lines

The previous equation is the **homogeneous differential equation of the initial heat balance**.

The solution, with the hypothesis of $\Delta\theta = \theta - \theta_a$ and $\Delta\theta_i = \theta_i - \theta_a$, is as follows:

$$\Delta\theta(t) = \Delta\theta_i e^{-\frac{t}{\tau}}$$



Thermal phenomena in lines

If, in the initial heat balance equation, we consider **phenomena so rapid that they cannot give rise to heat exchanges between the line's conductor and the environment (i.e., *adiabatic heat transfer*)**, the energy balance equation is as follows:

$$RI^2 dt = \gamma v c \; d\theta$$

This equation is only valid when the **duration of the phenomenon under examination is well below the time constant, namely** $\Delta t << \tau$. **This condition applies, in practice, to short circuits interrupted within a very short time (i.e., $t < 100 \div 500$ ms) thanks to protection systems.**

Thermal phenomena in lines

If we integrate the preceding equation and consider **the constant current value during the period in which the thermal phenomenon is studied**, we obtain:

$$I^2 \Delta t = \frac{\gamma v c}{R} \left(\theta_c - \theta_a\right)$$

Where θ_c is the temperature attained at the end of the period Δt and θ_a is the initial temperature.

In the case that the current is not constant during the period Δt, the value of the current can be replaced by its **effective value** during the period:

$$I = \sqrt{\frac{1}{\Delta t} \int_0^{\Delta t} i^2 \, dt}$$

Thermal phenomena in lines

The term $\int_0^{\Delta t} i^2\,dt$ is called the **Joule integral** or **thermal impulse.**

To understand the physical meaning of the **Joule integral**, we should consider the energy dissipated during the period Δt with a resistance R and a current i

$$W = \int_0^{\Delta t} R i^2\,dt \rightarrow \frac{W}{R} = \int_0^{\Delta t} i^2\,dt$$

Therefore, **the Joule integral is the energy per unit of resistance dissipated by the current i during the period Δt. It is also called specific energy.**

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Criteria for steady-state conductor sizing

Maximum current for bare conductors

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Criteria for steady-state conductor sizing

In thermal equilibrium, the following equation is valid:

$$RI^2 = K_t S_d \left(\theta_c - \theta_a\right)$$

where

- K_t is the heat transfer coefficient between the conductor and the environment
- S_d is the heat exchange surface area
- θ_c is the temperature of the conductor
- θ_a is the ambient temperature.

If we **neglect the skin effect** (i.e., consider the DC value of the electrical resistance) we obtain:

$$\frac{\rho l}{A} I^2 = K_t l P \left(\theta_c - \theta_a\right)$$

where

- A is the cross-section of the conductor
- P is the perimeter of the conductor

Criteria for steady-state conductor sizing

The maximum current I_z for a conductor is given by

$$I_z = \sqrt{\frac{PAK_t}{\rho}(\theta_c - \theta_a)}$$

This value depends on:

- **geometric** parameters of the conductor (cross-section A and perimeter P) ;
- characteristics of the **conductor material** (resistivity ρ);
- condition of **heat release to the exterior** (global heat transfer coefficient K_t);
- $(\theta_c - \theta_a)$, **maximum permissible temperature difference.**

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Maximum current for bare conductors

In the case of **bare conductors** (**without insulation**), i.e., busbars, we can assume that the temperatures involved are θ_a=40°C and θ_c=70°C ($\Delta\theta$=30°C). If we determine the global heat transfer coefficient K_t , it is possible to evaluate the maximum current intensity of the conductor itself as a function of the conductor cross-section.

Maximum current for bare conductors

Maximum current values for conductors with circular cross-section, in **direct or alternating current.** We can see that for conductors with diameters of less than 16 mm, the **DC and AC maximum current values are the same**, whereas for larger diameter values, **the skin effect starts to become more important and the DC maximum current value is greater than the AC value.**

			Maximum current [A]	
Diameter [mm]	Cross-section [mm²]	Weight [kg/m]	DC	AC
3	7,07	0,063	40	40
4	12,57	0,112	50	50
5	19,64	0,175	75	75
6	28,27	0,252	95	95
7	38,48	0,343	120	120
8	50,27	0,447	140	14C
9	63,62	0,556	165	165
10	78,54	0,699	185	185
12	113,10	1,007	235	235
14	153,94	1,370	285	285
16	201,06	1,789	350	345
18	254,47	2,265	420	410
20	314,16	2,796	485	475
22	380,13	3,383	560	540
24	452,39	4,026	630	610
25	490,87	4,369	660	630
28	615,75	5,480	780	740
30	706,86	6,291	860	820

The penetration coefficient is $a = \sqrt{\dfrac{2\rho}{\omega\mu}}$

with ρ=1.78 10^{-8} Ω/m, ω=314 s^{-1}

μ_0=4π10^{-7} H/m → a=9.5mm

Maximum current for bare conductors

Maximum current values for conductors with **rectangular cross-section**, alternating current.

Dimensions [mm]	Cross-section [mm^2]	Weight [kg/m]	Maximum current [A]		
			1 rod	2 rods	3 rods
10x2	20	0,178	85		
12x2	24	0,214	100		
15x2	30	0,267	115		
20x2	40	0,356	150		
10x3	30	0,267	105		
12x3	36	0,320	130		
15x3	45	0,401	165		
20x3	60	0,534	200		
25x3	75	0,668	235		
30x3	90	0,801	280		
40x3	120	1,068	355		
20x4	80	0,712	235		
25x4	100	0,890	280		
30x4	120	1,068	300		
40x4	160	1,424	385		
50x4	200	1,730	455		
20x5	100	0,890	280	515	710
25x5	125	1,113	334	600	830
30x5	150	1,335	365	635	930
40x5	200	1,780	450	775	1140
50x5	250	2,225	555	1020	1420
60x5	300	2,670	640	1190	1640
80x5	400	3,560	830	1530	2125
100x5	500	4,450	1035	1900	2650
40x6	240	2,140	510	940	1315
50x6	300	2,670	605	1110	1560
60x6	360	3,204	710	1305	1830
30x6	480	4,272	1010	1875	?530
100x6	500	5,340	1130	.2460	2920
40x8	320	2,848	585	1075	1510
50x8	400	3,560	705	1295	1820
60x8	480	4,272	830	1525	2410
80x8	640	5,696	1185	2180	3060
100x8	800	7,120	1300	2390	3350
120x8	960	8,544	1515	2810	3910
60x10	600	5,340	940	1730	2410
80x10	800	7,120	1285	2350	3200
100x10	1000	8,900	1450	2670	3710
120x10	1200	12,820	1715	3150	4390

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Maximum current for cables in steady-state conditions

For cables covered with **insulating material, the maximum temperature reached by the conductor will be equal to the temperatures of the first layers of insulation** → the proper functioning of the insulation depends heavily on the **operating temperature.**
Cable isolation materials are subject to **chemical reactions that modify their electrical characteristics and lead to progressive degradation**. The speed at which these reactions occur, which is zero at room temperature, increases exponentially with temperature according to **Arrhenius' law**

$$R = R_0 e^{-\frac{B}{KT}}$$

where:

- R is the speed of reaction;
- B is the specific activation energy of the chemical reaction;
- K is Boltzmann's constant;
- T is the absolute temperature in Kelvin (K).

Maximum current for cables in steady-state conditions

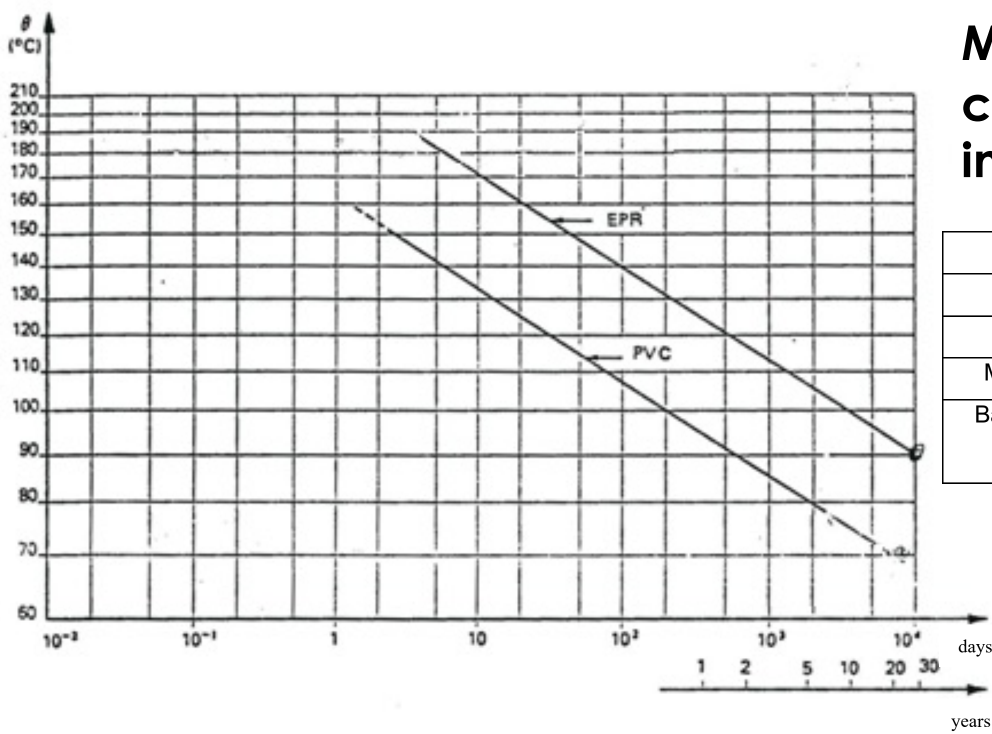
By analyzing the equation, we can see that **if the operating temperature increases, then the reaction rate increases and so do the chemical reactions**. This results in **higher degrdation rates for higher operating temperatures**.
It is important to remember that **cable life is inversely proportional to reaction speed**, as this reduces the life of the insulation.
It is possible to evaluate the **thermal life of the insulation** using the following expression:

$$\ln L = A + \frac{B}{T}$$

where:
- L is the lifetime of the insulation → the length of time for which the insulator can continuously maintain a specific temperature value without unacceptable degradation of its electrical and mechanical characteristics;
- A and B are specific constants calculated based on Arrhenius' law.

Maximum current for cables in steady-state conditions

If we establish an economically attractive service life for the insulation using the **Arrhenius chart**, the corresponding temperature (θ_s) is the **maximum service temperature**.
The figure shows thermal life curves for **PVC** and ethylene-propylene rubber – **EPR**.



Maximum service temperatures θ_s of conductors for cables with different insulation materials

Insulator	Maximum service temperature °C
PVC	Conductor: 70
XLPE and EPR	Conductor: 90
Mineral with PVC sheathing or bare	Metal sheath: 70
Bare mineral not in contact with other combustible materials	Metal sheath: 105 (nota 2)

Maximum current for cables in steady-state conditions

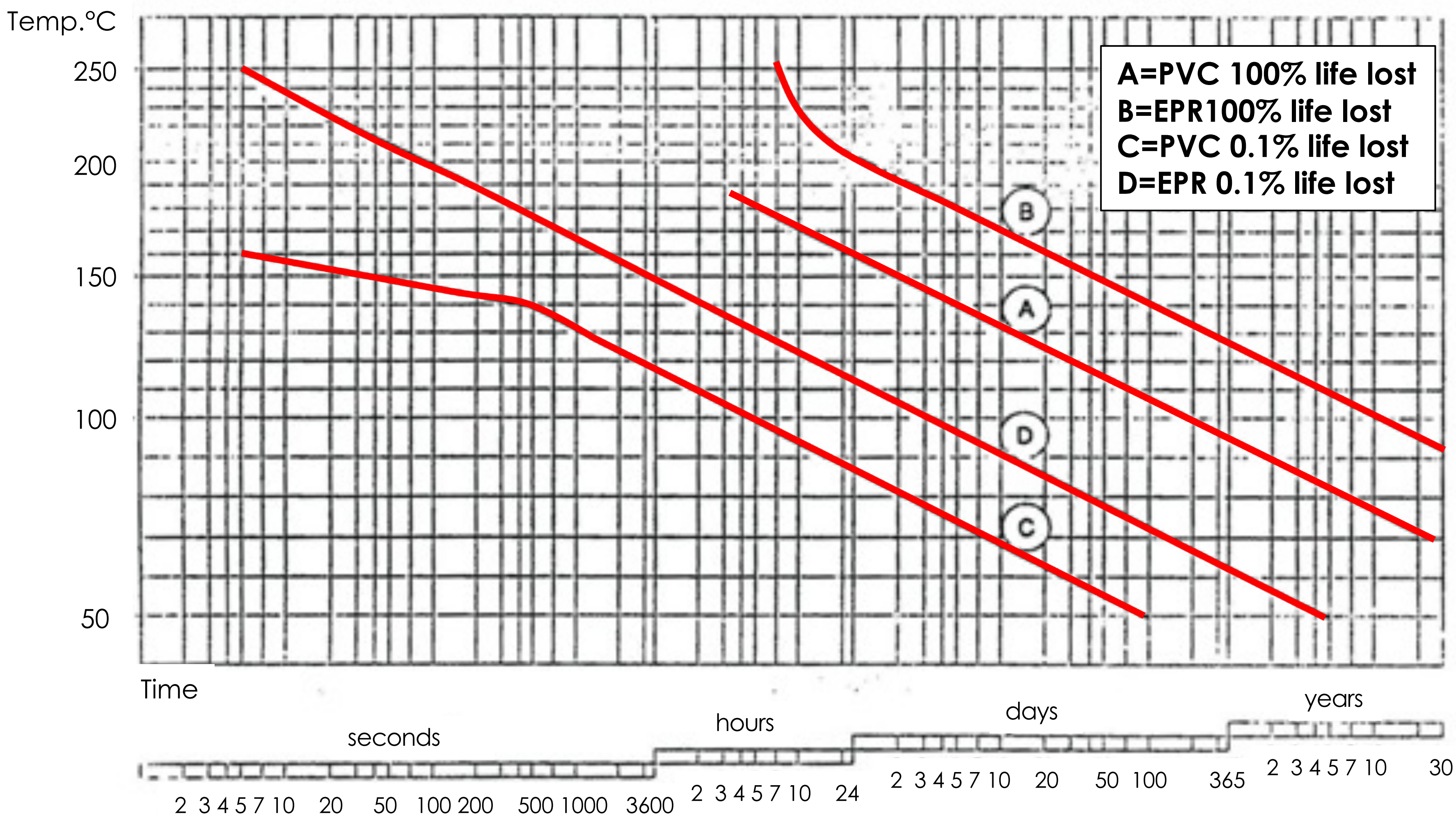
A cable carrying an electric current I_z reaches the temperature θ_s on the conductor. Each time the cable is traversed by a current with a greater value than I_z (therefore $\theta > \theta_s$), it leads to a **reduction in the cable's service life.**

Observations

- an overload is a **temporary event**;
- the duration of the overload t^* **may be shorter or longer than the time required to reach the operating temperature corresponding to the overload current**: to **increase the safety margin** we consider that **the overload brings the cable to the temperature θ_r of thermal regime for the entire duration of the overload**;
- Conventionally, we **accept a 10% loss in cable life** for all overloads that may occur during the cable's lifetime (20 years);
- Conventionally, it is acceptable to assume that a **single overload event can lead to a reduction in service life equal to 0.1% of the expected service life.**

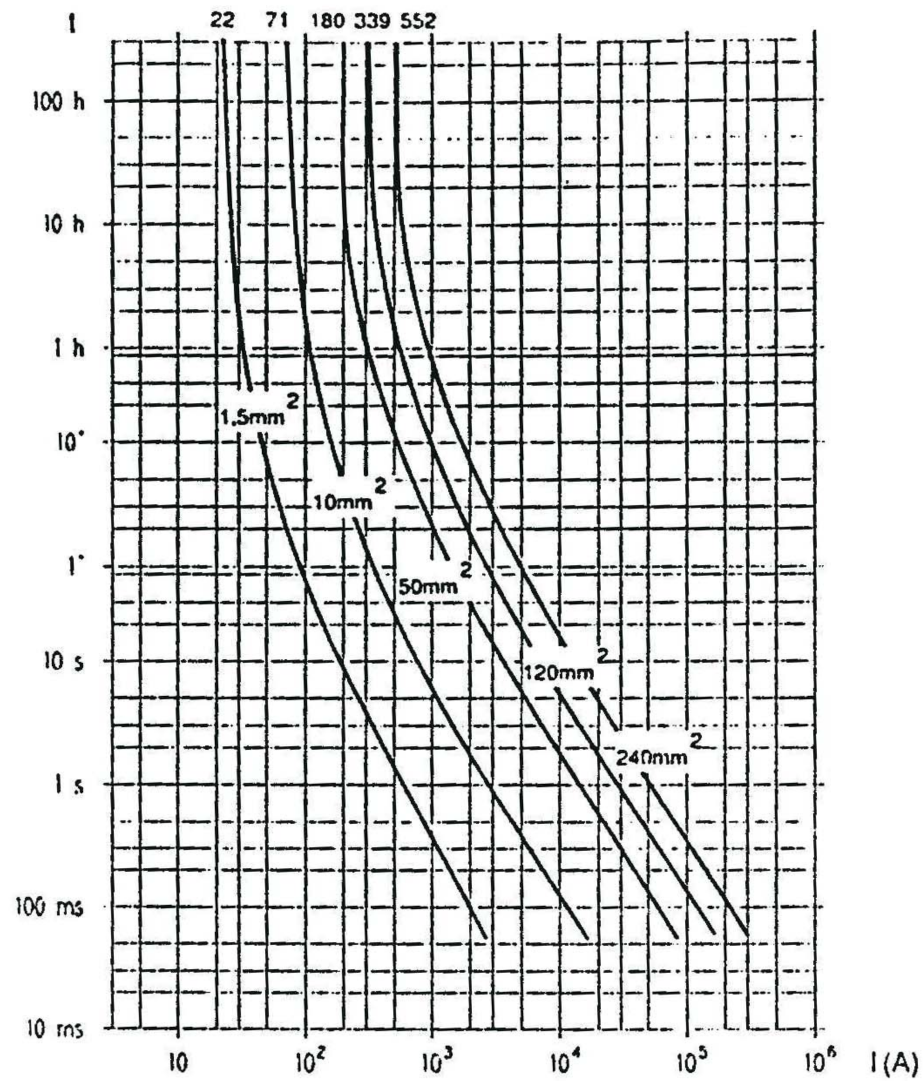
Maximum current for cables in steady-state conditions

Service life curves for insulating materials PVC (curve A) and EPR (curve B), 1/1000 service life reduction curves for PVC (curve C) and EPR (curve D) cables.



Maximum current for cables in steady-state conditions

Time-current curves for EPR / copper cables, three-core, laid in air.



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Adiabatic thermal phenomena in the event of a short-circuit

If the **duration of the thermal phenomenon is much shorter than the time constant (adiabatic heat transfer)**, i.e., $\Delta t << \tau$, the heat balance equation is as follows:

$$rI^2 dt = \gamma c v d\theta$$

If Δt is the protection tripping time and θ_b and θ_e are the temperatures at $t = 0$ and $t = \Delta t$, we have

$$I^2 \Delta t = \frac{\gamma c v}{r}(\theta_e - \theta_b)$$

If the conductor has the volume $v = lA$ (l length and A cross-section) and the resistance $r = \rho \frac{l}{A}$

Then:

$$I^2 \Delta t = \frac{\gamma c v}{r}(\theta_e - \theta_b) = K^2 A^2$$

where

$$K = \sqrt{\frac{\gamma c}{\rho}(\theta_e - \theta_b)}$$

Adiabatic thermal phenomena in the event of a short-circuit

$$K = \sqrt{\frac{\gamma c}{\rho}(\theta_e - \theta_b)}$$

The factor K depends on the following parameters:

- **physical constants of cable materials**, for example **density** (γ), **specific heat** (c) and **resistivity** (ρ);
- temperature at the **beginning of the event** (θ_b);
- temperature at the **end of the event** (θ_e);

Observation: during the cable sizing phase, the maximum permissible temperature will depend on the characteristics of the insulating material.

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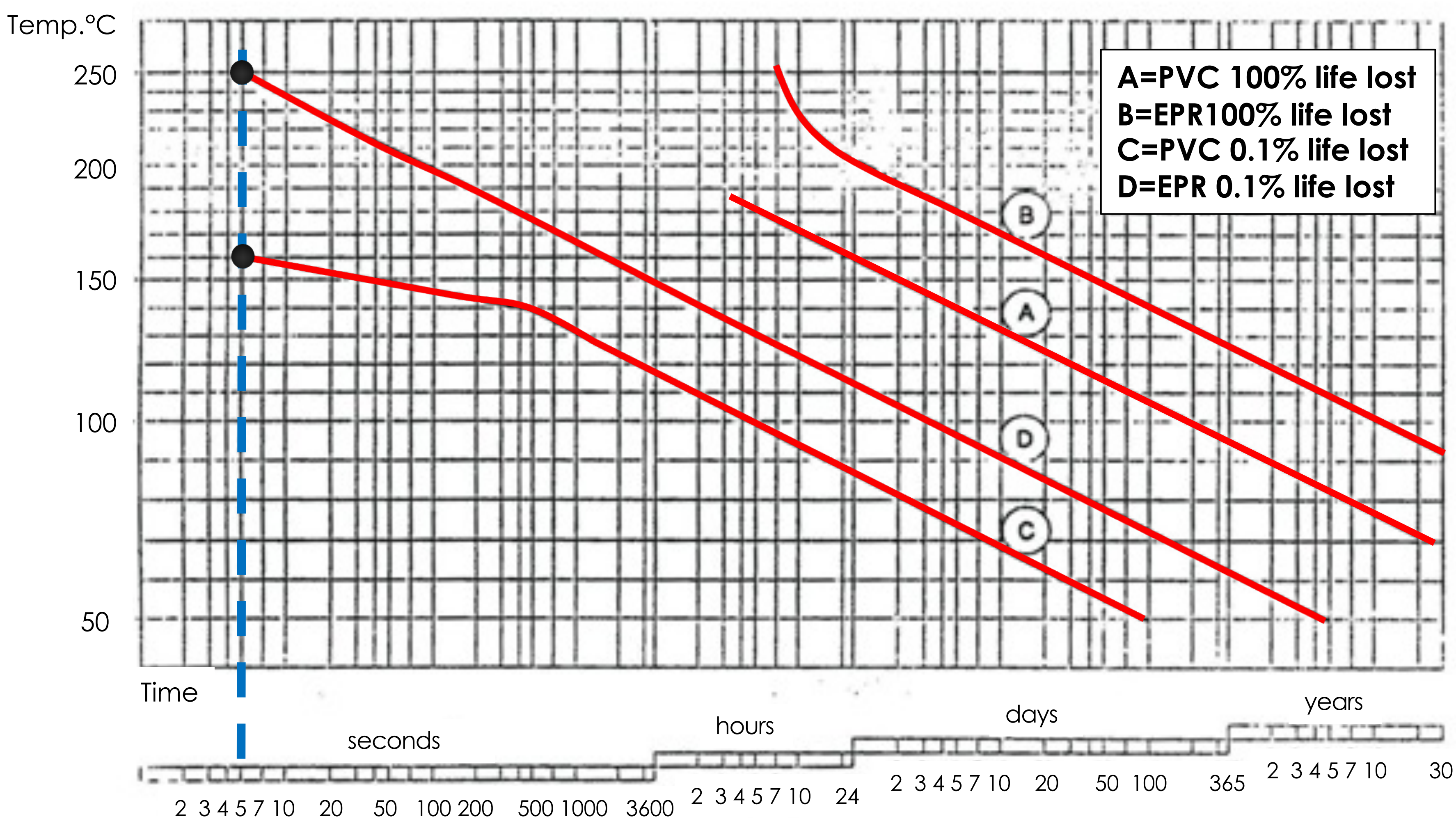
Permissible cable temperature in the event of a short-circuit

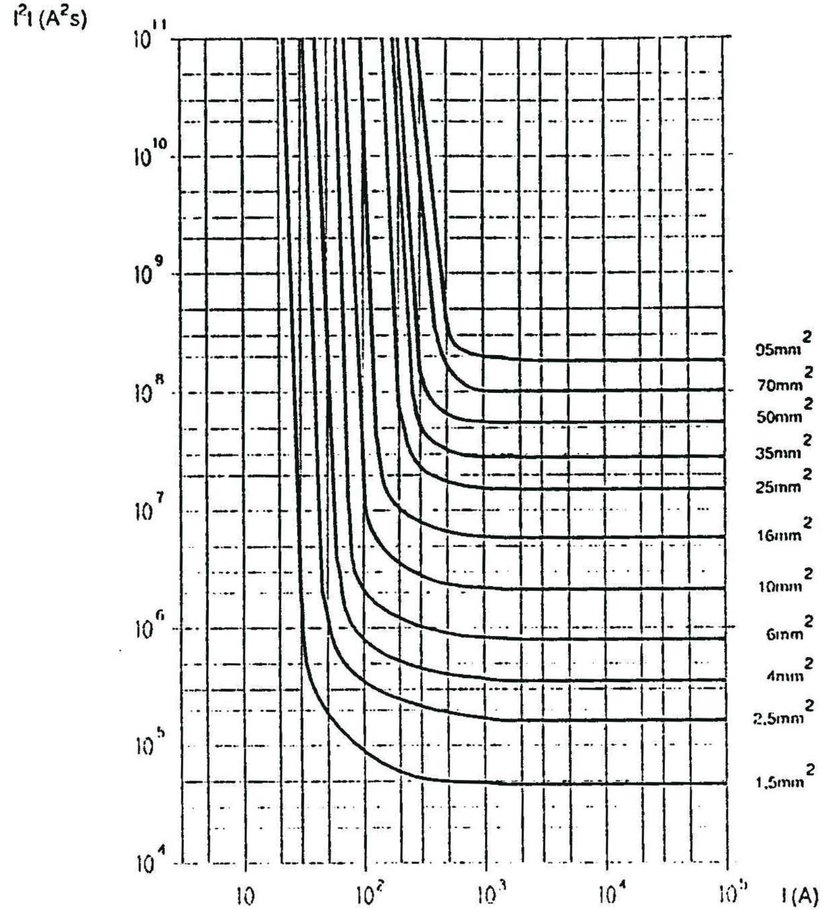
The temperature limit for cable conductors in the event of a short-circuit is 160 °C for PVC and 250°C for EPR (each short-circuit is assumed to have a maximum duration of 5s)



Permissible cable temperature in the event of a short-circuit

Joule integral (or specific energy) (I^2t) as a function of the **current** of the cable during a short-circuit.

Observation: the **vertical asymptote arrives at the maximum permissible current intensity**, and for this current value the specific energy entering the cable can have an infinite value, since in any case heat generation is compensated by heat release at operating temperature.
The horizontal asymptote shows the maximum permissible specific energy for different insulators and different cable cross-sections.

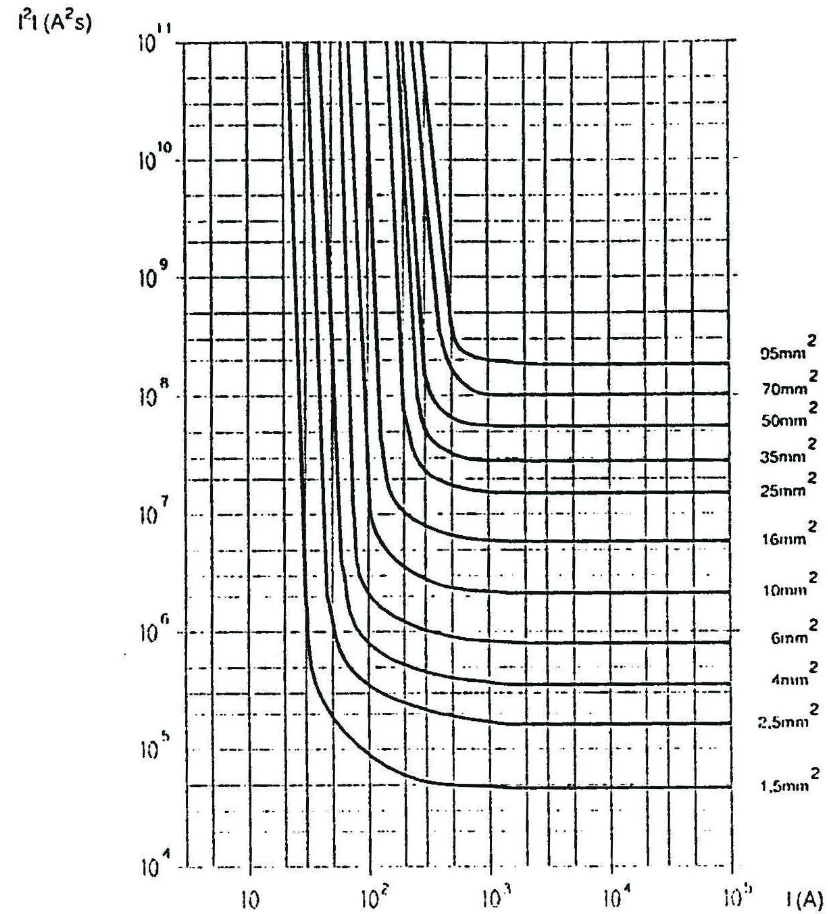


Joule integral (or specific energy) curves as a function of cable current (EPR, copper, three-core, laid in air)

Permissible cable temperature in the event of a short-circuit

In practice, the value of the horizontal asymptote corresponds to the product K^2A^2 where A is the cross-section of the cable and K a constant which depends on the insulation type (i.e., for PVC-insulated copper conductors $K=115$, while for EPR or XLPE $K=143$).

The value of K is always relative to an initial temperature θ_b equal to operating temperature θ_s (70°C for PVC, 90°C for EPR).



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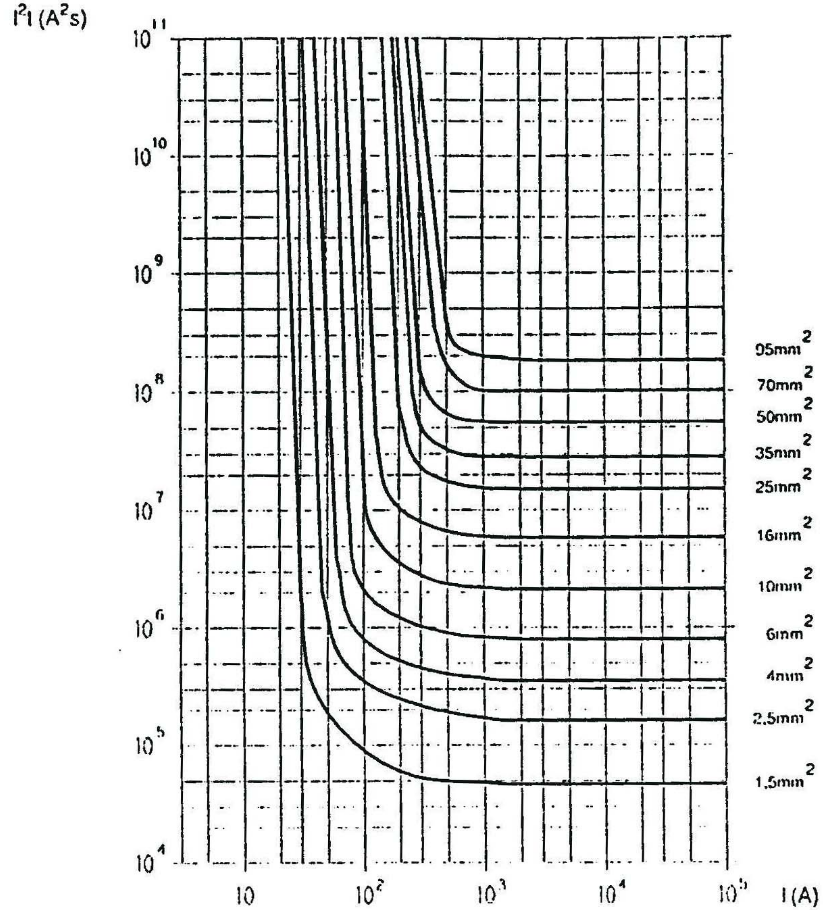
Permissible cable temperature in the event of a short-circuit

Protections must therefore intervene within a timeframe in which the associated energy that passes before the protection is triggered, equal to I^2t, is less than the maximum energy admissible by the cable in the form of a heat pulse equal to K^2A^2.



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